Perception and quality choice in vertically differentiated markets

Abstract

Consumers have bounded perception of goods’ quality in a vertically differentiated duopoly, treating sufficiently sufficient goods as homogeneous. More extreme product differentiation is observed, and firms must coordinate on equilibrium. Perception is endogenized by allowing firms to influence the ease of product comparison via firms’ presentation of their goods. It is shown that firms will act to improve consumer perception to the greatest extent possible.

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1 Introduction

That consumers’ perception of goods may be imperfect is readily observable. Consider Fortnum and Mason’s, who in error sold standard quality caviar costing around £40/kg as higher quality caviar costing around £1300/kg. No consumer perceived a difference.\(^1\)

Perception of stimuli is studied in psychophysics, a sub-discipline of psychology (see e.g. Falmagne (2002) and Weber (2004)), where there is a long tradition of studying how dissimilar stimuli must be for a difference to be detectable. Here, this tradition is applied to an economic context: When the qualities of goods are sufficiently similar, consumers do not perceive the difference and treat them as homogeneous. This article focuses, however, not on the individual choices of consumers with bounded perception, but rather on the market outcome when such consumers interact with profit maximizing firms.

The actions of firms to influence perception is also studied. The way in which a firm presents a good can have a large impact on the ease with which it can be compared to other goods. It is examined whether competition causes firms to facilitate such comparisons or hinder them.

It is found that firms must differentiate themselves to a greater extent, shifting their qualities further apart than the equilibrium in the baseline case of perfect perception. There is not a unique way of allocating this shift between firms, leading to a continuum of equilibria. The effect on profits compared to the baseline is ambiguous: There are equilibria in which bounded perception leads to greater profits for both firms, one firm and neither firm. This is due to on the one hand firms having to choose excessively high quality (which is costly) or excessively low quality (which consumers do not value) and on the other hand to greater differentiation meaning less intense price competition.

Consumer perception is endogenized by allowing each firm to present their goods to either help or hinder comparison. It is found that firms engage in behaviour to make it as easy

as possible for consumers to distinguish between similar goods. The market induces firms to make consumers’ decisions as easy as possible, despite potentially greater profits with bounded perception.

Consumers’ decision making in this article is founded in Rubinstein (1988)’s notion of a similarity relation, a binary relation on a set describing when its elements are treated by an individual as identical or non-identical. Similarity relations were originally proposed as a way of resolving the Allais paradox and other deviations from expected utility theory in lottery choice (see also Azipurua et al. (1993), Leland (1994), Buschena and Zilberman (1999) and related work on semi-orders by Luce (1956) and Fishburn (1970)). Leland (2002) uses a similar approach to explain intertemporal choice anomalies. However, these studies focus on individual decision making, rather than profit maximizing firms interacting with boundedly rational consumers.

Webb (2015) studies a similar model to the current article, but with sequential entry of firms into the market in order to study the effect of bounded perception on market concentration.

The influence of salience and attention has been a topic of recent study by, amongst others, Bordalo et al. (2012, 2013a, 2013b, 2013c, in press), Kőszegi and Szeidl (2013) and Cunningham (2013), in whose models individuals give greater weight in decision making to options which “stand out” in the choice set. The current study is in the same spirit, but examines a qualitatively different perceptual bias, focusing instead on how perception is affected by the similarity of goods’ attributes in the choice set.

Several experimental studies have examined the ease of comparison of goods by varying the degree of complexity with which they are presented: The more complex goods are, the harder it is to perceive the difference between them. Kalayci and Potters (2011) find that firms prefer to complicate the presentation of their goods to make it harder to distinguish between them. Crosetto and Gaudeul (2012) find that consumers prefer to have offers that are easy to compare, and that firms which make their goods easy to understand make greater
profit.

Sitzia and Zizzo (2011) find limited evidence that product complexity can induce consumers to buy greater quantities of a good, and Kalayci (2015) finds that greater complexity in goods’ prices rather than goods’ composition induces firms to set higher prices.

The importance of consumers’ perception has long been recognized in the field of marketing, for example Chandon and Ordabayeva (2009) examine how packaging shape can bias individuals’ assessment of volume. There has been investigation into the influence of labeling on consumer choice (for example Kwortnik et al. (2006)), which has also expanded into the question of whether nutritional labels induce consumers to make healthier choices (Variyam, 2008).

The specific economic institution considered here is a vertically differentiated product market (Shaked & Sutton, 1982), in which competing firms sell a good with heterogeneous levels of some objectively measurable quality. Consumers with perceptual limitations are introduced to a model similar to that in Motta (1993). This model serves as a tractable baseline against which the effects of perceptual limitations may be judged. The vertical differentiation framework is well suited to yield insights into the effects of bounded perception, as the ability of firms to distinguish the attributes of the goods they produce is instrumental to their ability to make a profit.

Section 2 describes the decision making process of consumers with bounded perception. Section 3 then develops and discusses a duopoly model of vertical differentiation when consumers have bounded perception. In section 4, perception is endogenized by allowing firms to influence it via the presentation of their goods. Section 5 concludes.

2 Bounded perception

Let \( c = (q, -p) \) represent paying price \( p \) to consume a good of quality \( q \). If \( Q = [0, \infty) \) is the set of qualities and \( P = [0, \infty) \) is the set of prices then the consumption set is \( C = Q \times -P. \succcurlyeq_u \)
is a weak preference relation on $C$ satisfying the usual assumptions. Let $\sim_s$ be a Rubinstein similarity relation (Rubinstein, 1988) on $Q$. $q \sim_s q'$ signifies that $q$ and $q'$ are sufficiently similar that they cannot be distinguished, whereas $q \sim_s q'$ signifies that they can. Let $\sim_s : Q \to Q$ be the $q' \in Q$ such that $q \sim q'' \forall q'' \in [q, q')$ and let $\sim_s : Q \to Q$ be the $q' \in Q$ such that $q \sim q'' \forall q'' \in (q', q]$. It is assumed (i) that $q \leq q \leq q$, (ii) that $q$ and $\bar{q}$ exist for every $q \in Q \setminus \{0\}$, and (iii) that $q$ and $\bar{q}$ are continuous and strictly increasing in $q$.

Together $\succsim_u$ and $\sim_s$ induce a decision preference relation $\succsim_d$ on $C$ in the following way:

(i) If $q \sim_s q'$ and $c \succsim_u c'$, $c \succsim_d c'$.

(ii) If $q \sim_s q'$ and $p \leq p'$, $c \succsim_d c'$.

$\succsim_u$ can be thought of as a “true” underlying preference relation and $\succsim_d$ as the preference relation used in decision making given the inability to distinguish between the qualities of some goods captured by $\sim_s$. Note that, although $\succsim_d$ is complete, it is generally not transitive, for example if $q \sim_s q'$, $p \leq p'$ then $c \succsim_d c'$ and if $q' \sim_s q''$, $p' \leq p''$ then $c' \succsim_d c''$, yet if $q \sim_s q''$ and $q < q''$ then it is possible to have $c'' \succsim_d c$.

Let $u = \alpha q - p$, $\alpha \in \mathbb{R}_+$ be a utility function representing the preference relation $\succsim_u$. Let $\delta \in [1, \infty)$ be the perception threshold. Then $\bar{q} = \delta q$ and $\bar{q} = \frac{\alpha}{\delta}$, so that $q \sim_s q'$ iff $\frac{\alpha}{\delta} \in \left(\frac{1}{\delta}, \delta\right)$.

The interpretation of the decision making mechanism adhered to in this article is that individuals are unable to perceive sufficiently small differences in quality. If two goods $q_h$ and $q_l$, $q_h \geq q_l$ with $\frac{q_h}{q_l} \geq \delta$, the qualities are far enough apart that the goods are perceived as heterogeneous. If, however, the quality ratio lies below the threshold ($\frac{q_h}{q_l} < \delta$) then the goods are similar enough that consumers perceive them to be homogeneous. The perceived quality of both in this case is assumed to be $q' = \lambda q_h + (1 - \lambda) q_l$, $\lambda \in [0, 1]$.\footnote{Note that, although it is natural to specify that the perceived quality is a weighted average of the high and low qualities, this is not a necessary assumption, and is made chiefly for expositional purposes.} If the qualities of the goods are similar enough, when they are presented to an individual together, the context causes her perception of them to be “blurred” and she sees each as having a quality which is some weighted average of the true quality.
As the absolute values of the goods’ quality increase, so does the absolute difference required for them to be seen as dissimilar. This is in agreement with a large body of perceptual research in psychology (see e.g. Falmagne (2002)) indicating that the perceived intensity of a stimulus is logarithmically proportional to the physical intensity, also known as the Weber-Fechner law.

The perception threshold is sharply discontinuous: an individual perfectly perceives the qualities of goods when $\frac{q_h}{q_i} = \delta$, yet reducing $q_h$ by an infinitesimal amount leads to them being seen as homogeneous. This may be rationalized by considering consumers to receive some noisy signal of quality $q + \epsilon$. Goods are then perceived as homogeneous if the ratio of the signal falls below the threshold.\(^3\) The discontinuous threshold is “smoothed” and perception of quality is imperfect, even when the quality ratio is above the threshold, leading to a much more natural perception mechanism. However, provided the signals of quality are not excessively noisy, results are qualitatively unchanged. Thus for clarity and tractability, the simplification of perfect perception of qualities when their ratio is above the threshold is made.

The quality of a good is measured as a single attribute. For the majority of goods, however, there are many different attributes. Quality in this case may be regarded as the single attribute which is the principle source of product differentiation. Another way to consider the single dimension of quality is as an aggregate measure of the quality of many attributes. As such an aggregate measure makes direct comparison more difficult, such an interpretation also makes bounded perception of quality more plausible.

Perception is not the sole interpretation consistent with such consumer behaviour, it could also be explained as an attentional bias. If the qualities of goods are sufficiently similar, then the price difference appears especially salient. Consumer attention is drawn away from quality and directed towards price, thus their purchase decision is based solely on price, similar to the inattentive consumers in Armstrong and Chen (2009).

\(^3\)I.e. the Rubinstein similarity relation is defined on the set of signals.
No similarity relation is introduced for price: consumers can perceive even a minuscule difference in price. This asymmetry is in contrast to the attentional mechanisms of, for example, Bordalo et al. (2012) or Kőszegi and Szeidl (2013), which treat all goods’ attributes symmetrically. A similarity relation could be introduced for prices, so that very small differences in prices may not be incorporated into individuals’ decision making, as in Bachi (2014). However, the range of prices similar enough that individuals treat them as identical is assumed to be much smaller than the corresponding range for qualities. The quality of a good is generally a much more nebulous, harder to detect attribute than its price, which is easy to compare to another good’s price, even when the difference is minimal.

3 Vertically differentiated duopoly

Having specified the behaviour of perceptually bounded consumers, it is possible to examine their interaction with profit maximizing firms. As the perception threshold is an intrinsic psychological mechanism, consumers’ reactions to given qualities and prices are fully determined, and so the following is a game played between firms. A vertically differentiated market, similar to the model in Motta (1993) will now be developed. Initially the baseline case of $\delta = 1$ (perfect perception) is given, with this then being compared to the case of $\delta > 1$, i.e. consumers with bounded perception.

3.1 Baseline case; $\delta = 1$

The model and results from Motta (1993) are presented below with a brief derivation found in section A.1 of the appendix.

Two identical firms, 1 and 2, produce a good with quality $q_i \in Q$ sold at price $p_i \in P$, $i \in \{1, 2\}$. Firms have fixed costs of quality $q_i^2$. Marginal costs of production are normalized to be 0, as this greatly improves tractability whilst leaving results qualitatively unchanged.

Consumers may purchase a single unit of the good, from which their utility is given
by \( u = \alpha q - p \). The payoff from the outside option of not consuming is normalized to 0. Consumers perceive any \( q > 0 \) as distinct from the outside option. \( \alpha \) represents a consumer’s taste for quality, and there is a unit mass of consumers with \( \alpha \) uniformly distributed in the interval \([0, 1]\).

The timing of the game is as follows:

**Period 1:** Firms simultaneously choose qualities, incurring fixed costs of quality.

**Period 2:** Having observed the chosen qualities, firms set prices, consumers purchase and firms earn revenue.

Let \( h \) always denote the firm producing the higher quality good, \( h \in \{1, 2\} \) and \( \ell \) the firm producing the lower quality good, \( \ell \in \{1, 2\} \), \( h \neq \ell \). This implies in equilibrium that \( p_h > p_\ell \). The consumer with taste parameter \( \alpha' = \frac{p_h - p_\ell}{q_h - q_\ell} \) is indifferent between high and low quality, and the consumer with \( \alpha'' = \frac{p_\ell}{q_\ell} \) is indifferent between low quality and not consuming. Demand for firm \( h \) is \( 1 - \alpha' \) and demand for firm \( \ell \) is \( \alpha' - \alpha'' \). The timing of the game is as follows:

**Period 1:** Firms simultaneously choose qualities, incurring fixed costs of quality.

**Period 2:** Having observed the chosen qualities, firms set prices, consumers purchase and firms earn revenue.

Given qualities \( q_h \) and \( q_\ell \), equilibrium prices in period 2 are

\[
\begin{align*}
    p_h^* &= 2q_h \left( \frac{q_h - q_\ell}{4q_h - q_\ell} \right), \\
    p_\ell^* &= q_\ell \left( \frac{q_h - q_\ell}{4q_h - q_\ell} \right) 
\end{align*}
\]

so that firms choose qualities in the first period to maximize

\[
\begin{align*}
    \pi_h (q_h, q_\ell) &= \frac{4q_h^2 (q_h - q_\ell)}{(4q_h - q_\ell)^2} - \frac{1}{2} q_h^2, \\
    \pi_\ell (q_h, q_\ell) &= \frac{q_h q_\ell (q_h - q_\ell)}{(4q_h - q_\ell)^2} - \frac{1}{2} q_\ell^2. 
\end{align*}
\]

Standard equilibrium qualities \( q_h^* \) and \( q_\ell^* \) are
\[ q_h^* = \mu^* \left( \frac{4\mu^* - 7}{(4\mu^* - 1)^3} \right), \quad q_l^* = \mu^*^2 \left( \frac{4\mu^* - 7}{(4\mu^* - 1)^3} \right) \]  

(3)

where

\[ \mu^* = \frac{q_h^*}{q_l^*} \]  

(4)

is the ratio of the high quality to the low quality in equilibrium. This ratio is constant and can be shown to be equal to the unique real solution of \(4\mu^3 - 23\mu^2 + 12\mu - 8 = 0\), which is approximately \(\mu^* \approx 5.251\), implying from equation (3) that \(q_h^* \approx 0.253\) and \(q_l^* \approx 0.048\). Equilibrium profits are \(\pi_h^* \approx 0.0244\) and \(\pi_l^* \approx 0.00153\).

### 3.2 Bounded perception; \(\delta > 1\)

Now let all consumers have an identical perception threshold of \(\delta > 1\). Given that for \(\frac{q_h}{q_l} < \delta\) consumers perceive the two goods as homogeneous, both having quality \(q'\), they will either purchase the good with the lower price, or not purchase. They will never purchase the good with the higher price.

That consumers never purchase the good with the higher price leads to a key result.

**Proposition 1.** (i) In equilibrium, firms will never choose qualities such that \(\frac{q_h}{q_l} < \delta\), i.e., consumers will always be able to perceive a difference between the firms’ equilibrium qualities.

(ii) If \(\delta > \mu^*\), a necessary condition for equilibrium is that \(\frac{q_h}{q_l} = \delta\).

All proofs are contained in the appendix. It was specified that, when \(\frac{q_h}{q_l} < \delta\), consumers perceived both goods as having a quality \(q' = \lambda q_h + (1 - \lambda) q_l\), \(\lambda \in [0, 1]\). However, it should be emphasized that the only necessary restriction for this result on consumers’ behaviour when \(\frac{q_h}{q_l} < \delta\) is that they never buy the higher priced good.

The intuition behind part (i) is straightforward: If firms choose qualities which consumers
treat as homogeneous, they end up in the Bertrand trap and make a loss. As they can make 0 by choosing \( q = 0 \), this will never be observed in equilibrium.

As \( \mu^* \) is the ratio of \( q_h \) to \( q_\ell \) in the baseline case, \( \delta > \mu^* \) implies firms will no longer produce the baseline qualities, but choose qualities which are further apart. Part (ii) then states that in equilibrium they will never produce qualities such that \( \frac{q_h}{q_\ell} > \delta \). The intuition for this is that if \( \frac{q_h}{q_\ell} > \delta \), firms’ profits and best response functions are locally as in the baseline case. Thus, as in the baseline case, each firm will wish to choose a quality closer to its rival’s.

This is not the case when \( \frac{q_h}{q_\ell} = \delta \). Here, it is possible for firms to wish to move closer together in the baseline case, but to be prevented from doing so with \( \delta > \mu^* \) by the prospect of consumers perceiving the goods as homogeneous. Hence, it must be a necessary condition for equilibrium that \( \frac{q_h}{q_\ell} = \delta \).

When the baseline equilibrium no longer holds, equilibrium qualities may then be found by imposing restrictions on qualities satisfying \( \frac{q_h}{q_\ell} = \delta \) such that no firm wishes to deviate. The following six conditions provide an exhaustive list of the possible ways each firm can deviate, and so if all six hold, then the firms must be in equilibrium:

(i) Firm \( h \) does not choose \( q_h = 0 \), making \( \pi_h = 0 \). \hfill (Fh0)

(ii) Firm \( \ell \) does not choose \( q_\ell = 0 \), making \( \pi_\ell = 0 \). \hfill (F\ell0)

(iii) Firm \( h \) does not produce \( q_h > \delta q_\ell \) above the perception threshold. \hfill (FhT)

(iv) Firm \( \ell \) does not produce \( q_\ell < \frac{q_h}{\delta} \) below the perception threshold. \hfill (F\ellT)

(v) Firm \( h \) does not undercut firm \( \ell \) by producing \( q_h < q_\ell \). \hfill (FhU)

(vi) Firm \( \ell \) does not leapfrog firm \( h \) by producing \( q_\ell > q_h \). \hfill (F\ellL)

The conditions are explicitly derived in the appendix, and it is shown that Fh0, F\ell0 and F\ellL are redundant. Equilibrium can then be stated.
Proposition 2. In equilibrium, firms choose qualities

\[
q_h^* = \begin{cases} 
\mu^* \left( \frac{4\mu^* - 7}{(4\mu^* - 1)^3} \right) & \text{if } \delta \leq \mu^* \\
\in [FhT(\delta), \min \{F\ell T(\delta), FhU(\delta)\}] & \text{if } \delta > \mu^* 
\end{cases}
\]

\[
q_\ell^* = \begin{cases} 
\mu^* \left( \frac{4\mu^* - 7}{(4\mu^* - 1)^3} \right) & \text{if } \delta \leq \mu^* \\
\in \frac{1}{\delta} [FhT(\delta), \min \{F\ell T(\delta), FhU(\delta)\}] & \text{if } \delta > \mu^* 
\end{cases}
\]

where

\[
FhT(\delta) = \frac{4\delta (4\delta^2 - 3\delta + 2)}{(4\delta - 1)^3} 
\]

\[
F\ell T(\delta) = \frac{\delta^3 (4\delta - 7)}{(4\delta - 1)^3} 
\]

\[
FhU(\delta) = \frac{2\delta^3 (4\delta^2 - 1)}{(4\delta - 1)^2 (1 + \delta + \delta^2 + \delta^3)}. 
\]

Equilibrium exists for any finite \( \delta \geq 1 \).

For \( \delta \leq \mu^* \), equilibrium is unchanged from the baseline case. For \( \delta > \mu^* \) there is a continuum of equilibria, which is illustrated in figure 1.

Firms’ equilibrium profits are obtained by substituting \( q_h^* \) and \( q_\ell^* \) into equation (2). Expressions for profits are not given explicitly for reasons of brevity, however it can be shown that

Proposition 3. For \( \delta > \mu^* \), firm h (\( \ell \)) makes a greater profit than in the baseline case if inequality (A.8) (inequality (A.9)) is fulfilled, and less profit than in the baseline case if it is not.
Inequalities (A.8) and (A.9) are not reproduced in the main text due to their length, however they are illustrated in figure 1.

It is also possible to compare equilibria for consumers with a given threshold. The comparative statics are given in terms of $q^*_h$. Note that since $\frac{q^*_h}{q^*_\ell} = \delta$, for fixed $\delta$ an increase in $q^*_h$ is equivalent to an increase in $q^*_\ell$. It implies the absolute levels of quality in the market are increased.

**Proposition 4.** In equilibrium, given some $\delta > \mu^*$:

(i) Firm $h$’s profit is decreasing in $q^*_h$ and firm $\ell$’s profit is increasing in $q^*_h$.

(ii) Consumer surplus is decreasing in $q^*_h$.

(iii) If $\mu^* < \delta \leq \delta_{TS} \approx 7.359$, total surplus is increasing in $q^*_h$, where $\delta_{TS}$ is defined as the unique positive root of equation (A.11). If $\delta > \delta_{TS}$, total surplus is maximized at

$$q^*_{TS} = \frac{\delta^2 (12\delta^2 - \delta - 2)}{2(\delta + 1)(4\delta - 1)^2}$$

below which it is increasing in $q^*_h$ and above which it is increasing in $q^*_\ell$.

$q^*_{TS}$ is illustrated in figure 1.

### 3.3 Discussion

Having derived equilibrium for consumers with bounded perception, it can be seen that the perception threshold $\delta$ must exceed $\mu^* \approx 5.251$ for there to be any difference to the baseline case. Therefore it is worth examining whether such values of $\delta$ could be reasonably expected to be observed, and so whether bounded perception is relevant.

Psychophysical research suggests that the minimum perceivable difference in intensity of some stimulus (equivalent to $\delta - 1$) typically takes values much smaller than the values $\delta$ must take to have an impact on quality choice. This *just noticeable difference* has been estimated.
to be 0.079 for brightness, 0.048 for loudness and 0.02 for heaviness (Techtsoonian, 1971), yet here $\delta - 1 \gtrsim 4.251$ for consumers’ bounded perception to be relevant.

However, this is due to the necessity of $\delta$ being greater than the standard equilibrium quality ratio $\mu^*$, and $\mu^*$ depends heavily on the distribution of consumers in the economy and the assumption of 0 marginal cost. Both a more concentrated distribution of consumers and a positive marginal cost of production would lead to a lower $\mu^*$, and thus a lower perception threshold would be required to influence quality choice.

Another possibility is to regard $q$ as excess quality over some minimum required level $q_{\text{min}}$, so that the ratio of high to low quality is now $\frac{q_h + q_{\text{min}}}{q_h + q_{\text{min}}}$. Thus if $q_{\text{min}} = 5$, for example, then the threshold needed for perception to be relevant falls to $\delta - 1 \gtrsim 0.041$, which is in line with the psychophysical results quoted above.

There is now a continuum of equilibria. The reason is that when $\delta > \mu^*$ firms must increase the ratio of high to low quality in order to avoid the Bertrand trap. However, there is no unique way of doing this: a given shift in the ratio of qualities can be split many ways between the firms. For $\delta > \mu^*$, there is a range of qualities such that $\frac{q_h}{q_{\ell}} = \delta$ and such that in the standard case the only profitable deviation for a firm is to produce a quality closer to its rival’s. When consumers have bounded perception, such a deviation now results in a loss, leading to a range of qualities for which there is no profitable deviation.

Firms are presented with a coordination game, in that they must choose qualities such that $\frac{q_h}{q_{\ell}} = \delta$, but must coordinate on the absolute values that satisfy this ratio.

Proposition 3 compares the different equilibria and finds that firm $h$ prefers low absolute values of quality and firm $\ell$ prefers high absolute values of quality. Thus the firms’ problem is not one of pure coordination: there is no salient equilibrium which both firms prefer.

To see why higher levels of quality have different effects for each firm, note that if one firm were granted a monopoly, it would choose $q = 0.25$. Duopoly forces one firm to choose above and one firm below the quality it would ideally prefer. Hence higher levels of quality

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4This value is easily derived from equation (A.3a) by setting $q_{\ell} = 0$. 

13
imply firm $h$ is further away from its ideal quality, reducing profit, whereas firm $\ell$ is closer, increasing profit.

It is possible to compare firms' profits to the baseline as well. Proposition 3 shows the effect of bounded perception is ambiguous: compared to the baseline, bounded perception may increase both firms', one firm's or neither firms' profit. This is due to the ratio of high to low quality being greater than in the baseline for $\delta > \mu^*$, which has two contrasting effects. Firstly, price competition in period 2 is reduced when firms differentiate themselves more. For a given quality choice, the further away the rival firm's quality, the higher the price a firm may charge before consumers switch to the rival (or to not consuming). On the other hand, firms must make suboptimal quality choices. Increasing quality is costly for firm $h$, and decreasing quality reduces willingness-to-pay for firm $\ell$, which tends to reduce profit. This has the opposite effect to reduced price competition, and for each firm, either effect may dominate.

Proposition 4 states that consumers prefer high levels of quality. A caveat to this result is that it holds strictly in a comparative static sense. That it is possible to say anything definite regarding consumer surplus is due to the fact that in equilibrium consumers perceive both goods distinctly. Consumers' perceived surplus depends on the context of the choice set they have. Thus when measuring the change in consumer surplus if consumers shift from one equilibrium to another, it is necessary to account for the new goods being viewed in context with the old. It is not clear that a consumer is made better off by consuming a higher quality good if she pays a higher price for it, yet cannot perceive the new good as superior to the old.

The principle result of this section is that the more bounded consumers’ perception of quality, the greater the degree of product differentiation. Given that products are differentiated in the baseline, albeit to a lesser degree, the behaviour of firms is qualitatively unchanged. To search for empirical examples of bounded perception, it is necessary to compare markets across sectors or over time.
Comparing different markets, the quality of a product may be assessed more easily in one than the other. This could be due to a fundamental difference in the nature of the quality of different goods, or due to a clear scale by which to measure quality being available in one market but not the other. The prediction of this model is then that products will be differentiated to a greater degree in markets in which product quality is hard to assess than in markets in which it is easy to assess.

It can be reasonably supposed that when a new type of product is introduced, the perception threshold is high due to consumers’ inexperience. As they grow more accustomed to it, the threshold is reduced. Thus it should be expected that for new product types, a large degree of product differentiation is observed, followed by less differentiation over time.

Bounded perception does not necessarily improve over time, however. In many markets price comparison websites operate. These facilitate comparing firms on prices, but quality information is often much more difficult to assess. In the airline industry, for example, the advent of flight search websites has lead to people to focus far more on the price of a journey than the airline’s service level. This has lead to success for budget airlines and those with the highest level of service, but problems for more traditional firms who charge more than budget airlines but struggle to differentiate their level of service.

It was stated in section 2 that the one-dimensional quality measure used here could be interpreted as an aggregate measure of several attributes. It is reasonable to assume that the greater the number of attributes that make up this measure, the more difficult the direct comparison of goods is and the higher the perception threshold. Thus the model predicts that in markets with more complex goods, a greater degree of differentiation should be observed then for markets with simple goods.

The current section describes equilibrium in a market with bounded perception taking the perception threshold as exogenous. This allows the threshold to be endogenized in the following section, in which the actions of firms to increase or decrease the ease with which consumers perceive their goods are described.
4 Endogenous perception threshold

So far it has been assumed that the perception threshold is exogenous. However, firms can often influence the perception of their goods. This could either have the effect of lowering the threshold, for example if firms coordinated on a clear colour scheme allowing quality to be discerned at a glance, or by the printing of relevant information prominently on a good’s packaging, or it could raise the threshold if the low quality firm designs its packaging to ape that of the high quality good, or if information is hidden in the fine print.

4.1 Model

To endogenize the perception threshold, suppose each firm may exert presentational effort \( s \) to alter the perception threshold, with \( s_i \in [\underline{s}, \overline{s}], \underline{s} < 0, \overline{s} > 0, i \in \{h, l\} \). As methods to influence perception are generally, as in the examples above, rooted in design and presentation, this effort is considered to be costless. Given each firm’s presentational effort, the perception threshold becomes \( D(s_T, \delta) \), where \( s_T = s_1 + s_2 \) and \( D(s_T, \delta) \) has the following properties:

(i) \( D(s_T, \delta) \geq 1 \).

(ii) \( D(0, \delta) = \delta \).

(iii) If \( D(s_T, \delta) > 1 \) and \( s'_T > s_T \), then \( D(s'_T, \delta) > D(s_T, \delta) \).

(iv) If \( D(s_T, \delta) > 1 \) and \( \delta' > \delta \), then \( D(s_T, \delta') > D(s_T, \delta) \).

\( \delta \) can thus be thought of as a “default” threshold when neither firm exerts any particular effort to manipulate perception. Negative \( s_T \) implies improved consumer perception and positive \( s_T \) indicates worsened perception relative to the default. The threshold is symmetric with respect to firms’ presentational efforts.

Endogenizing bounded perception via presentational effort means effectively that firms can choose the frame in which consumers view their goods, as in Piccione and Spiegler (2012) and Spiegler (2014).
Two extra conditions must now be fulfilled in equilibrium:

(vii) Firm $h$ does not wish to choose a different presentational effort. \hspace{1cm} (Fh$\delta$)

(viii) Firm $l$ does not wish to choose a different presentational effort. \hspace{1cm} (F$l$$\delta$)

Let $D = D(2\$h, \delta)$ be the lowest threshold obtainable for a given $\delta$. Then

**Proposition 5.** In equilibrium, firm choose qualities and presentational efforts $q^*_h$, $s^*_h$ and $q^*_l$, $s^*_l$ given by

$$
q^*_h \begin{cases} 
= \mu^*^3 \left( \frac{4\mu^* - 7}{(4\mu^* - 1)} \right)^3 \\
\in [FhT(D), \min\{F\ell T(D), FhU(D)\}] 
\end{cases}
$$

where

$$s^*_h \begin{cases} 
= \$h \\
\in \{s_h : D(s_T, \delta) \leq \mu^*\} 
\end{cases}
$$

if $D \leq \mu^*$

$$
q^*_l \begin{cases} 
= \mu^*^2 \left( \frac{4\mu^* - 7}{(4\mu^* - 1)} \right)^3 \\
\in \frac{1}{\delta} [FhT(D), \min\{F\ell T(D), FhU(D)\}] 
\end{cases}
$$

where

$$s^*_l \begin{cases} 
= \$l \\
\in \{s_\ell : D(s_T, \delta) \leq \mu^*\} 
\end{cases}
$$

if $D > \mu^*$

(8a)

It is interesting to compare the comparative statics of firms’ ability to influence perception (i.e. $\$h$). Consider $\$h' > $ with $D' > \mu^*$. Then any $q^*_h$, $q^*_l$ that formed an equilibrium with $\$h$ will not form an equilibrium with $\$h'$, as it implies a different equilibrium ratio of qualities. However, it is always possible to find some $q^*_i$, $i \in \{h, \ell\}$ that firm $i$ produces in some equilibrium given both $\$h$ and $\$h'$, even though the quality produced by firm $j \neq i$ will differ. Comparative statics are then performed from the perspective of a firm producing the same quality $q^*_i$ given different $\$h$.

By taking this approach it is possible to show

**Proposition 6.** (i) Let firm $i \in \{h, \ell\}$ produce quality $q^*_i$ in equilibrium given $\$h$, earning profit $\pi^*_i(q^*_i, \$h)$. Then given $\$h' < $, conditional on producing the same quality $q^*_i$, $\pi^*_i(q^*_i, \$h') \leq \pi^*_i(q^*_i, \$h)$. 

17
(ii) Let firm $i \in \{h, \ell\}$ produce quality $q_i^*$ in equilibrium given $s$. Then given $s' < \bar{s}$, conditional on the firm producing the same quality $q_i^*$, consumer surplus is weakly higher.

### 4.2 Discussion

In section 3, although bounded perception caused firms to position themselves farther apart than in the baseline and they also has to coordinate on equilibrium, their behaviour was qualitatively unchanged. Here though, a prediction is made that firms will engage in novel behaviour: both low and high quality firms will present their goods so as to make perception of their quality as easy as possible. The effect of a competitive environment on consumers’ decision making bias is to reduce it as much as possible.

It is not immediately obvious that firms should always choose to reduce, rather than exacerbate bounded perception, given that one or both firms may with $\delta > \mu^*$ find itself in an equilibrium earning greater profit than in the baseline. Furthermore, provided $\bar{s} \geq -\bar{s}$, a firm can always negate any presentational effort of the other firm to improve perception.

The intuition behind proposition 5 is that the equilibrium qualities for $\delta > \mu^*$ found in the previous section (equation (5)) form equilibria only due to bounded perception removing a profitable deviation. Without the perception threshold, each firm would prefer to produce a quality closer to its rival’s.

When firms are able to influence consumers’ perception, the profitable deviation is again present and firms opt to lower the perceptual threshold. Equilibrium then requires that such profitable deviations to produce a quality closer to its rival’s does not exist, which is the case only if $s_h = s_\ell = \bar{s}$ and it is not possible to further enhance perception, or the threshold is lowered sufficiently that the baseline equilibrium is reached.

Presentational effort was assumed to be costless. However, it is possible that the costs of such effort may be significant, for example if it involves a large advertisement campaign. Provided the cost is not excessive though, firms will still reduce consumers’ perceptual bias to some extent.
Examples of firms presenting their goods in such a way to ease consumer perception of them are readily available. In many countries, for example, dairy firms have coordinated on a colour-coding scheme for different types of milk. Such schemes, which make it easy for consumers to perceive the type of milk at a glance, occur even though they require sustained coordination between firms.

Another example is the carat measure of gold purity. The difference in the purity of different gold items is very difficult to assess. It is made much easier by the fact that jewelers have for many years coordinated on the universal use of an easy to understand scale.

Proposition 6 examines the welfare effects of firms’ ability to exert presentational effort. For a given equilibrium quality, a firm earns less profit the more firms are able to improve consumer perception. This is due to the resultant lower perception threshold meaning the rival firm choosing a quality closer to its own, and thus in the price setting stage the competition being more fierce. For consumers, on the other hand, the opposite is true, as they benefit from the increased price competition and consumer welfare increases.

A caveat to the welfare results is that, due to the multiplicity of equilibria, an exogenous increase in $s$ is not guaranteed to decrease profit of improve consumer welfare.

The result that, rather than trying to exploit their ability to confuse consumers, firms instead try to eliminate consumers’ biases is markedly different to, for example, Gabaix and Laibson (2006). In that paper, competitive forces do not act to debias consumers, as such debiased consumers are not then profitable to firms. It also contradicts the experimental findings of Kalayci and Potters (2011), in which in contrast to this article, consumers were homogeneous in their taste for quality.

Aside from the specific results on biased perception, an illustration is thus also given on the complexities of boundedly rational consumers interacting with profit maximizing firms. The outcomes may be vastly different depending on the particular form of bounded rationality being considered, and also on relatively minor changes to the market structure. Thus any potential market intervention should be carefully considered before enactment.
5 Conclusion

It is impossible to perceive a small enough difference between two products. This article has taken this simple observation and shown that it can have significant influence on market outcomes, in terms of the qualities and prices observed in equilibria and profit, the effect on which is ambiguous.

There are many extensions that may be made to the simple framework used here. For example, repeated interactions would be a fruitful avenue of research, both if consumers’ perception of quality increases with repeated consumption, and if firms have an incentive to reduce quality over time when consumers perceive the good they buy today as the same as the good they bought yesterday.

There is a growing recognition that perceptual issues should not be lightly disregarded in both empirical and theoretical work, and this has been again demonstrated in this study.
Appendix

A.1 Derivation of standard equilibrium

Firms profits are

\[ \pi_h(q_h, q_\ell) = p_h \left( 1 - \frac{p_h - p_\ell}{q_h - q_\ell} \right) - \frac{1}{2} q_h^2, \quad \pi_\ell(q_h, q_\ell) = p_\ell \left( \frac{p_h - p_\ell}{q_h - q_\ell} - \frac{p_\ell}{q_\ell} \right) - \frac{1}{2} q_\ell^2. \]  

(A.1)

The first order conditions are

\[ \frac{\partial \pi_h(q_h, q_\ell)}{\partial q_h} = 1 - 2 \frac{p_h - p_\ell}{q_h - q_\ell} \quad \text{(A.2a)} \]
\[ \frac{\partial \pi_\ell(q_h, q_\ell)}{\partial q_\ell} = \frac{p_h - 2 p_\ell}{q_h - q_\ell} - \frac{2 p_\ell}{q_\ell} \quad \text{(A.2b)} \]

from which it can be seen that the second order conditions are negative. Solving the first order conditions for \( p_h \) and \( p_\ell \) results in the prices in equation (1).

The first and second order conditions of equation (2) are

\[ \frac{\partial \pi_h(q_h, q_\ell)}{\partial q_h} = \frac{4q_h (4q_h^2 - 3q_h q_\ell + 2q_\ell^2)}{(4q_h - q_\ell)^3} - q_h, \quad \frac{\partial^2 \pi_h(q_h, q_\ell)}{\partial q_h^2} = -\frac{q_h^2 (40q_h + 8q_\ell)}{(4q_h - q_\ell)^4} - 1 \quad \text{(A.3a)} \]
\[ \frac{\partial \pi_\ell(q_h, q_\ell)}{\partial q_\ell} = \frac{q_\ell^2 (4q_h - 7q_\ell)}{(4q_h - q_\ell)^3} - q_\ell, \quad \frac{\partial^2 \pi_\ell(q_h, q_\ell)}{\partial q_\ell^2} = -\frac{2q_\ell^2 (8q_h + 7q_\ell)}{(4q_h - q_\ell)^4} - 1. \quad \text{(A.3b)} \]

Using the substitution \( q_h = \mu q_\ell, \mu \geq 1 \), the first order conditions may be rearranged to become

\[ \frac{4\mu (4\mu^2 - 3\mu + 2)}{(4\mu - 1)^3} = \mu q_\ell, \quad \text{(A.4a)} \]
\[ \frac{\mu^2 (4\mu + 7)}{(4\mu - 1)^3} = q_\ell. \quad \text{(A.4b)} \]

Taking the ratio of these and rearranging yields \( 4\mu^3 - 23\mu^2 + 12\mu - 8 = 0 \), the solution to which is \( \mu^* \), and equation (A.4b) then gives the equilibrium qualities in equation (3).
A.2 Proof of proposition 1

(i) If $\frac{q_h}{q_\ell} < \delta$, Bertrand competition with effectively homogeneous goods and identical marginal costs of 0 for each firm occurs. Firms earn no revenue and make a loss for any $q_h, q_\ell > 0$. Then as each firm can make 0 profit from selecting 0 quality, $\frac{q_h}{q_\ell} < \delta$ cannot be an equilibrium. Any $q_h > 0$ is perceivably different to $q_\ell = 0$, so that $q_h = 0$ is not a best response to $q_\ell = 0$. For any $q_h > 0$ it is possible to find some $0 < q_\ell \leq \frac{q_h}{\delta}$ which is strictly positive and perceivably different to $q_h$. From the first order condition of $\pi_\ell (q_h, q_\ell)$, $\frac{\partial \pi_\ell (q_h, q_\ell)}{\partial q_\ell} \bigg|_{q_\ell=0} > 0$, so $q_\ell = 0$ is not a best response to any $q_h = 0$. □

(ii) Let $\delta > \mu^*$. For $\frac{q_h}{q_\ell} > \delta$, firms’ profits are given by equation (2), and thus their best responses (given firm $h$ ($\ell$) is constrained to produce the higher (lower) quality) are determined by the first order conditions (equation (A.3)). The unique solution to $\frac{\partial \pi_h (q_h, q_\ell)}{\partial q_h} = 0$, $\frac{\partial \pi_\ell (q_h, q_\ell)}{\partial q_\ell} = 0$ is $q_h^*, q_\ell^*$, with $\frac{q_h^*}{q_\ell^*} = \mu^*$. Thus if $\delta > \mu^*$, firms cannot be in equilibrium for $\frac{q_h}{q_\ell} > \delta$ and so $\frac{q_h}{q_\ell} = \delta$ is a necessary condition for equilibrium. □

A.3 Proof of proposition 2

Firstly it is shown that the baseline equilibrium is the unique equilibrium for $\delta \leq \mu^*$. Let $\delta \leq \mu^*$. For any $\frac{q_h}{q_\ell} > \delta$, firms’ best response functions are as in the standard case and so the only equilibrium given this is $q_h^*, q_\ell^*$. By proposition 1, $\frac{q_h}{q_\ell} < \delta$ will not be an equilibrium. If $\frac{q_h}{q_\ell} = \delta$, the only potentially profitable deviation for firm $h$ ($\ell$) is to produce a higher (lower) quality, so necessary conditions for equilibrium are $\frac{\partial \pi_h (q_h, q_\ell)}{\partial q_h} \leq 0$ and $\frac{\partial \pi_\ell (q_h, q_\ell)}{\partial q_\ell} \geq 0$. Substituting $q_h = \delta q_\ell$ into the first order conditions, a necessary condition for $\frac{\partial \pi_h (q_h, q_\ell)}{\partial q_h} \leq 0$ and $\frac{\partial \pi_\ell (q_h, q_\ell)}{\partial q_\ell} \geq 0$ is $4\delta^3 - 23\delta^2 + 12\delta - 8 > 0$. The only real root of this polynomial is $\delta = \mu^*$, so for $\delta \leq \mu^*$, $\frac{q_h}{q_\ell} = \delta$ will only be observed in equilibrium if $\delta = \mu^*$ and $q_h = q_h^*, q_\ell = q_\ell^*$.

Secondly, equilibrium for $\delta > \mu^*$ is found. By proposition 1 it must be that $\frac{q_h}{q_\ell} = \delta$. Substituting $q_h = \delta q_\ell$ into equation (2) gives
\[ \pi_h = \frac{4q_h \delta (\delta - 1)}{(4\delta - 1)^2} - \frac{1}{2}q_h^2 \]  
(A.5a) \[ \pi_\ell = \frac{q_h (\delta - 1)}{(4\delta - 1)^2} - \frac{q_h^2}{2\delta^2} \]  
(A.5b)

**Equilibrium conditions**

**Condition \( Fh0 / F\ell0 \)**

A firm deviates to produce 0 quality if it otherwise makes a loss. Equating equation (A.5a) (equation (A.5b)) to 0 and rearranging gives condition \( Fh0 \) \((F\ell0)\) as \( q_h \leq Fh0 (\delta) = \frac{8\delta(\delta-1)}{(4\delta-1)^2} \)

\( q_h \leq F\ell0 (\delta) = \frac{2\delta^2(\delta-1)}{(4\delta-1)^2} \).

**Condition \( FhT / F\ellT \): Firm \( h/\ell \) above/below threshold**

For firm \( h \) to wish to deviate to produce above the threshold it must be that the first order condition of \( \pi_h \) is positive given \( \frac{q_h}{q_\ell} = \delta \). Equating equation (A.3a) to 0 and rearranging gives condition \( FhT \) as \( q_h \leq FhT (\delta) = \frac{4\delta(4\delta^2 - 3\delta + 2)}{(4\delta - 1)^2} \). Similarly firm \( \ell \) deviates to below the threshold if the first order condition of \( \pi_\ell \) is negative given \( \frac{q_h}{q_\ell} = \delta \). Equating equation (A.10b) to 0 and rearranging gives condition \( F\ellT \) as \( q_\ell \geq F\ellT (\delta) = \frac{\delta^2(4\delta^2 - 7)}{(4\delta - 1)^2} \).

**Condition \( FhU \): Firm \( h \) undercut**

Let \( h = 1 \) so that firm 1 is the high quality firm and firm 2 is the low quality firm. Assume firm 1 undercuts by choosing \( q_1^{uc} \) such that \( \frac{q_2}{q_1^{uc}} = \delta \), i.e. the highest quality below \( q_2 \) such that consumers perceive the goods as distinct. Substituting \( q_h = q_2 = \frac{q_2}{\delta} \) and \( q_\ell = q_1^{uc} = \frac{q_2}{\delta^2} \) into equation (2b) gives \( \pi_1 = \frac{(\delta-1)}{\delta(4\delta-1)^2}q_1 - \frac{q_2^2}{2\delta^2} \). Subtracting its profit from deviating from its profit in the candidate equilibrium, equating this to 0 and rearranging then allows condition \( FhU \) to be given as \( q_1 \leq FhU (\delta) = \frac{2\delta^3(4\delta^2 - 1)}{(4\delta - 1)^2(1+\delta+\delta^2+\delta^3)} \).

The assumption that firm 1’s best response conditional on undercutting is \( q_1 = \frac{q_2}{\delta} \) is valid only if the first order condition of \( \pi_\ell \) evaluated at \( q_h = q_2 = \frac{q_2}{\delta} \) and \( q_\ell = q_1^{uc} = \frac{q_2}{\delta^2} \) is non negative. Substituting the relevant qualities into equation (A.3b) shows the first order
condition is \( \frac{\partial \pi_1}{\partial q_1} = \frac{\delta^2(4\delta^2-7)}{(4\delta-1)^3} - \frac{q_1}{\delta^2} \). This is linearly decreasing in \( q_1 \), so it is sufficient to show that it is non negative at the maximum \( q_1 \) satisfying \( q_1 \leq FhU(\delta) \). Substitution reduces the condition to \( 4\delta^5 - 3\delta^4 - 35\delta^3 + 5\delta^2 - 2 \geq 0 \). Numerically, this is not satisfied for \( \delta > \mu^* \) and \( FhU(\delta) \) is the condition for no undercutting. An analogous argument holds when firm 2 is the high quality firm.

**Condition \( F\ell L \): Firm \( \ell \) leapfrog**

Let \( \ell = 1 \) so that firm 1 is the low quality firm and firm 2 the high quality firm. Assume firm 1 leapfrogs to produce \( q_1^{le} \) such that \( \frac{q_1^{le}}{q_2} = \delta \), i.e. the lowest quality above \( q_2 \) that consumers perceive as heterogeneous. Substituting \( q_h = q_1^{le} = \delta^2 q_1 \) and \( q_\ell = q_2 = \delta q_1 \) into equation (2a) gives \( \pi_1 = \frac{4\delta^2(4\delta^2-1)}{(4\delta-1)^2} q_1 - \frac{\delta^2 q_1^2}{2} \). Substituting its profit from deviating from its profit in the candidate equilibrium, equating this to 0 and rearranging then allows condition \( F\ell L \) to be given as \( q_1 \geq F\ell L(\delta) = \frac{2\delta^2(4\delta^2-1)}{(4\delta-1)^2(1+\delta+\delta^2+\delta^3)} \).

The assumption that firm 1’s best response conditional on leapfrogging is \( q_1^{le} = \delta q_2 \) is valid only if the first order condition of \( \pi_h \) evaluated at \( q_h = q_1^{le} = \delta^2 q_1 \) and \( q_\ell = q_2 = \delta q_1 \) is non positive. Substituting the relevant qualities into equation (A.3a) shows the first order condition is \( \frac{\partial \pi_1}{\partial q_1} = \frac{4\delta(4\delta^2-3\delta+2)}{(4\delta-1)^3} - \delta q_1 \). This is decreasing in \( q_1 \), so it is sufficient to show it is non positive at the lowest \( q_1 \) satisfying \( q_1 \geq F\ell L(\delta) \). Substitution reduces the condition to \( -8\delta^5 + 6\delta^4 + 10\delta^3 + 5\delta^2 - 2\delta + 4 \leq 0 \). Numerically, this is satisfied for \( \delta > \mu^* \), so \( F\ell L(\delta) \) is the condition for no leapfrogging. An analogous argument holds when firm 2 is the low quality firm.

**Condition redundancy**

Conditions \( FhT \) and \( F\ell L \) set a lower limit on \( q_h \). Rearranging \( F\ell L(\delta) - FhT(\delta) \geq 0 \) results in \( 8\delta^5 - 14\delta^4 + 4\delta^3 + 5\delta^2 - 3\delta + 4 \geq 0 \), which is not satisfied for \( \delta > \mu^* \), and so \( FhT \) is the binding lower limit.

Conditions \( FhS \), \( F\ell S \), \( FhU \) and \( F\ell T \) set an upper limit on \( q_h \). \( F\ell S(\delta) - FhS(\delta) \geq 0 \) can
be reduced to \( \delta \geq 4 \) and \( FhS(\delta) - FhU(\delta) \geq 0 \) can be reduced to \( \delta \geq 2 \), so \( FhS \) and \( F\ell S \) are not binding for \( \delta > \mu^* \). \( F\ell T(\delta) - FhU(\delta) = 0 \) is reduced to

\[ 4\delta^4 - 35\delta^3 + 5\delta^2 + 5\delta - 9 = 0. \]  \tag{A.6} 

Numerically, the unique root of this equation satisfying \( \delta > 1 \) is \( \delta \approx 8.591 \). Then \( F\ell T(\delta) \) is binding for \( \mu^* < \delta \approx 8.591 \) and \( FhU(\delta) \) is binding for \( \delta \gtrsim 8.591 \).

An equilibrium exists for \( \delta > \mu^* \) if there exists some \( \delta \) satisfying \( FhT(\delta) \leq \delta \leq \min\{F\ell T(\delta), FhU(\delta)\} \). \( F\ell T(\delta) - FhT(\delta) = 0 \) may be rearranged to give to give \( 4\delta^3 - 23\delta^2 + 12\delta - 8 = 0 \), which has a single real solution at \( \delta = \mu^* \). Since, for example \( F\ell T(6) - FhT(6) \approx 0.049 \), \( F\ell T(\delta) - FhT(\delta) > 0 \) for \( \delta > \mu^* \). \( FhU(\delta) - FhT(\delta) = 0 \) may be rearranged to give \( 8\delta^5 - 6\delta^4 - 10\delta^3 - 5\delta^2 + 2\delta - 4 = 0 \), which numerically has a single real root at \( \delta \approx 1.706 < \mu^* \). Since, for example, \( FhU(10) - FhT(10) \approx 0.021 \), \( FhU(\delta) - FhT(\delta) > 0 \) for \( \delta > \mu^* \) and for \( \delta > \mu^* \) there exists some \( \delta \) satisfying \( FhT(\delta) \leq \delta \leq \min\{F\ell T(\delta), FhU(\delta)\} \).

□

### A.4 Proof of proposition 3

By substituting the equilibrium qualities into equation (2a), the ratio of equilibrium profit for \( \delta > \mu^* \) to profit in the baseline case is

\[
\frac{\pi_{h}^{\delta > \mu^*}}{\pi_{h}^{\text{base}}} = \frac{2 \left(4\mu^* - 1\right)^{6}}{\mu^4 \left(4\mu^* - 7\right) \left(8 - 40\mu^* + 39\mu^*^2 - 4\mu^*^3\right)} \frac{\left(4\delta (\delta - 1) - q_h^* - \frac{q_h^2}{2}\right)}{(4\delta - 1)^2}. \tag{A.7} 
\]
Equating this to 1 and exploiting the quadratic formula, firm $h$ makes greater profit than in the baseline if

$$\frac{4\delta (\delta - 1)}{(4\delta - 1)^2} - \sqrt{\frac{16\delta^2 (\delta - 1)^2}{(4\delta - 1)^4} - \frac{\mu^* (4\mu^* - 7) (8 - 40\mu^* + 39\mu^* - 4\mu^*)}{(4\mu^* - 1)^6}}$$

$$< q_h^* < \frac{4\delta (\delta - 1)}{(4\delta - 1)^2} + \sqrt{\frac{16\delta^2 (\delta - 1)^2}{(4\delta - 1)^4} - \frac{\mu^* (4\mu^* - 7) (8 - 40\mu^* + 39\mu^* - 4\mu^*)}{(4\mu^* - 1)^6}}$$

(A.8)

By the same method it is found that firm $\ell$ makes greater profit than in the baseline if

$$\frac{\delta^2 (\delta - 1)}{(4\delta - 1)^2} - \sqrt{\frac{\delta^4 (\delta - 1)^2}{(4\delta - 1)^4} - \frac{\delta^2 \mu^* (4\mu^* - 7) (2 - 10\mu^* + 15\mu^* - 4\mu^*)}{(4\mu^* - 1)^6}}$$

$$< q_\ell^* < \frac{\delta^2 (\delta - 1)}{(4\delta - 1)^2} + \sqrt{\frac{\delta^4 (\delta - 1)^2}{(4\delta - 1)^4} - \frac{\delta^2 \mu^* (4\mu^* - 7) (2 - 10\mu^* + 15\mu^* - 4\mu^*)}{(4\mu^* - 1)^6}}.$$ 

(A.9)

□

A.5 Proof of proposition 4

(i) The first order conditions of profit given $\frac{q_h}{q_\ell} = \delta$ are

$$\frac{\partial \pi_h}{\partial q_h} = \frac{4\delta (4\delta^2 - 3\delta + 2)}{(4\delta - 1)^3} - q_h$$

(A.10a)

$$\frac{\partial \pi_\ell}{\partial q_\ell} = \frac{\delta^2 (4\delta - 7)}{(4\delta - 1)^3} - q_\ell.$$

(A.10b)

From equation (A.3a), firm $h$’s profit is maximized at $q_h^{max} = \frac{4\delta (\delta - 1)}{(4\delta - 1)^2}$. Rearrangement of $FhT(\delta) - q_h^{max} \geq 0$ gives $2\delta + 1 \geq 0$, which holds for all $\delta \geq 1$, so for equilibrium values of $q_h$ and $\delta > \mu^*$, firm $h$’s profit is decreasing in $q_h$.

From equation (A.10b), firm $\ell$’s profit is maximized at $q_\ell^{max} = \frac{4\delta^3 (\delta - 1)}{(4\delta - 1)^2}$. Rearrangement of $q_\ell^{max} - F\ell T(\delta) \geq 0$ gives $2\delta + 1 \geq 0$ which holds for all $\delta \geq 1$. Rearrangement of
\( q_h^{\text{max}} - F h U(\delta) \geq 0 \) gives \( \delta^4 - 8\delta^3 + 2\delta - 1 \geq 0 \), which holds for \( \delta \gtrsim 7.970 \). As \( F h U(\delta) \) is only binding for \( \delta \gtrsim 8.591 \), for equilibrium values of \( q_h \) and \( \delta > \mu^* \), firm \( h \)'s profit is decreasing in \( q_h \).

(ii) Aggregate consumer surplus for \( \delta > \mu^* \) is

\[
CS = \int_{\alpha}^{1} (\alpha q_h - p_h) d\alpha + \int_{\alpha}^{\alpha'} (\alpha q_\ell - p_\ell) d\alpha,
\]

where \( \alpha' (\alpha'') \) is the taste parameter of the consumer indifferent between \( q_h \) and \( q_\ell \). After integration and the substitution \( q_\ell = \frac{\mu_\ell}{\delta} \) this becomes

\[
CS = \frac{\delta (4\delta + 5) q_h^2}{2(4\delta - 1)^2},
\]

so that \( \frac{\partial CS}{\partial q_h} > 0 \).

(iii) Rearranging \( \pi_h + \pi_\ell + CS \) gives the total surplus as

\[
TS = \frac{(12\delta^2 - 2) q_h}{2(4\delta - 1)^2} - \frac{2(2\delta + 1) q_h^2}{2\delta^2}.
\]

Taking the first order condition and rearranging gives equation (7).

\[
F h U(\delta) - q_h^{TS} \geq 0 \quad \text{is rearranged to give} \quad 4\delta^3 - 11\delta^2 - \delta + 2 \geq 0 \quad \text{which has the unique root satisfying} \quad \delta \geq 1 \quad \text{of} \quad \delta \approx 2.775, \quad \text{above which it holds, thus} \quad TS \quad \text{is decreasing for at least some equilibrium qualities of} \quad \delta > \mu^*.
\]

\[
q_h^{TS} - F \ell T(\delta) \geq 0 \quad \text{reduces to} \quad 16\delta^4 + 8\delta^3 - 55\delta^2 + 26\delta - 16 \geq 0,
\]

which has a single root satisfying \( \delta > 1 \) at \( \delta \approx 1.439 \), above which it holds, and so for \( \mu^* < \delta \lesssim 8.591 \) total surplus is increasing in equilibrium values of \( q_h \).

\[
q_h^{TS} - F h T(\delta) = 0 \quad \text{is reduced to}
\]

\[
8\delta^4 - 62\delta^3 + 24\delta^2 - 7\delta - 2 = 0.
\]

Let \( \delta_{TS} \approx 7.359 \) be the unique real root of this equation, so for \( \delta \leq \delta_{TS} \) Total surplus is increasing in equilibrium values of \( q_h \) and for \( \delta > \delta_{TS} \) it is maximized at \( q_h^{TS} \).

\[\square\]

### A.6 Proof of proposition 5

Consider some \( q_h, s_h, q_\ell, s_\ell \) such that \( \frac{q_h}{q_\ell} = D(s_T, \delta) > \mu^* \), \( s_h > s_\ell \) and conditions (i)-(vi) fulfilled. From the derivation of \( F h T(\delta) \), \( \frac{\partial \pi_h}{\partial q_h} < 0 \), so firm \( h \) wishes to set \( s_h' < s_h \), \( q < q_h \). This is the case for all \( s_h > s_\ell \), unless \( D(s_T, \delta) \leq \mu^* \). An analogous argument holds for firm \( l \): Consider some \( s_h, s_\ell \in \{ (s_h, s_\ell) : D(s_T, \delta) \leq \mu \} \). Then from the derivation of the standard equilibrium, the only qualities that form an equilibrium are \( q_h^*, q_\ell^* \). \[\square\]
A.7 Proof of proposition 6

(i) As $s' < s$ implies $D' \leq D$ it is sufficient to show that $\frac{\partial \pi_h^*}{\partial D} < 0$ and $\frac{\partial \pi_\ell^*}{\partial D}$.

Given some threshold $D$ and some equilibrium quality $q_h^*$, firm $h$ makes profit

$$\pi_h^* = \frac{4D(D - 1)}{(4D - 1)^2} q_h^* - \frac{1}{2} q_h^*$$  \hspace{1cm} (A.12)

from which it follows that $\frac{\partial \pi_h^*}{\partial D} = \frac{4(2D+1)}{(4D-1)^3} q_h^* > 0$. Similarly, given $D$ and $q_\ell^*$ firm $\ell$ makes profit

$$\pi_\ell^* = \frac{D(D - 1)}{(4D - 1)^2} q_\ell^* - \frac{1}{2} q_\ell^*$$  \hspace{1cm} (A.13)

so that $\frac{\partial \pi_\ell^*}{\partial D} = \frac{(2D+1)}{(4D-1)^3} > 0$.

(ii) It is sufficient to show that $\frac{\partial CS}{\partial D} < 0$ for a constant $q_h^*$ and $q_\ell^*$. Given some threshold $D$ and firm $h$ producing some equilibrium quality $q_h^*$, consumer surplus is $CS = \frac{D(4D+5)}{2(4D-1)^2} q_h^*$ so that $\frac{\partial CS}{\partial D} = -\frac{(28D+5)}{2(4D-1)^3} < 0$. Similarly, given $D$ and firm $\ell$ producing some equilibrium quality $q_\ell^*$, consumer surplus is $CS = \frac{4D+5}{2(4D-1)^2} q_\ell^*$. Thus $\frac{\partial CS}{\partial D} = -\frac{2(4D+11)}{(4D-1)^3} q_\ell^* < 0$. \hfill \Box
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Figure 1: Solid lines indicate values of $q_h$ which form an equilibrium. Dashed lines indicate whether firms earn more or less than in the baseline. The dot-dashed line shows the $q_h$ at which total surplus is maximized.